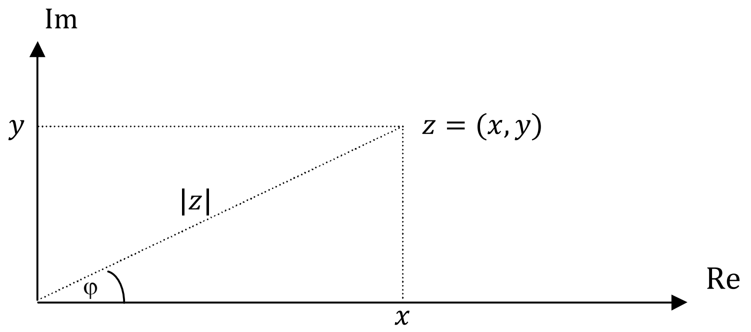
Liczby zespolone



$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

- *Postać algebraiczna:* $z = x + iy$ $\quad \bar{z} = x - iy$
- *Postać trygonometryczna:* $z = |z|(\cos\varphi + i\sin\varphi)$ $\quad \bar{z} = |z|(\cos\varphi - i\sin\varphi)$
- *Postać wykładnicza:* $z = |z|e^{i\varphi}$ $\quad \bar{z} = |z|e^{-i\varphi}$

$$z \cdot \bar{z} = (x + iy)(x - iy) = x^2 + y^2 = |z|^2$$

$$|z| = \sqrt{x^2 + y^2} \qquad \left.\begin{matrix} \cos\varphi = \frac{x}{|z|} \\ \sin\varphi = \frac{y}{|z|} \end{matrix}\right\} \Rightarrow \varphi = \cdots$$

Wzory de Moivre'a dla $\boxed{z = |z|(\cos\varphi + i\sin\varphi)}$:

- Potęgowanie:

$$z^n = |z|^n(\cos n\varphi + i\sin n\varphi)$$

- Mnożenie:

$$z_1 \cdot z_2 = |z_1| \cdot |z_2|(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2))$$

- Dzielenie:

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2))$$

- Pierwiastkowanie:

$$\sqrt[n]{z} = \omega_i \qquad \text{dla} \quad i = \{0, 1, \dots, n-1\}$$

$$\omega_0 = \sqrt[n]{|z|}(\cos\frac{\varphi}{n} + i\sin\frac{\varphi}{n})$$

$$\boxed{skok = \frac{2\pi}{n}}$$

$$\omega_1 = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi}{n} + skok\right) + i\sin\left(\frac{\varphi}{n} + skok\right)\right)$$

$$\omega_2 = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi}{n} + 2 \cdot skok\right) + i\sin\left(\frac{\varphi}{n} + 2 \cdot skok\right)\right)$$

$$\vdots$$

$$\omega_k = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi}{n} + k \cdot skok\right) + i\sin\left(\frac{\varphi}{n} + k \cdot skok\right)\right)$$

$$\vdots$$

$$\omega_n = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi}{n} + 2\pi\right) + i\sin\left(\frac{\varphi}{n} + 2\pi\right)\right) = \omega_0$$